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Number Theory and Educational Exchange in Hungary

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The purpose of this project was to spend a year studying number theory at the Alfréd Rényi Institute of Mathematics while promoting Hungarian-American educational exchange. The project consisted of coursework, independent research, and helping compile course notes for a number theory class taught in the Budapest Semesters in Mathematics exchange program. The report begins with background on the subject of analytic number theory, which was my focus, centering on recent developments and Hungarian contributions. The report also contains specifics of my academic tenure in Hungary as well as an overview of the course whose notes I am completing.

1. Introduction

Hungary has a knack for beautiful mathematics. The results of Pál Erdős cannot be overstated, but one should also not forget the other legendary mathematicians whose portraits line the main stairway in the humble orange building on Reáltanoda Street. It was a thrill to be among the great minds at the Rényi Institute and to have access to their advice and guidance. My time as a Fulbright Scholar has proven a great opportunity for me to develop as a mathematician and has influenced not only my abilities, but also the focus of my research. My stay at the Rényi Institute consisted mainly of studying a subfield of number theory known as analytic number theory. This report begins with background on this subfield with focus on major contributions by Hungarian mathematicians.

Apart from my studies in number theory, I was also affiliated with the Budapest Semesters in Mathematics program, helping Professor Csaba Szabó complete the course notes for his introductory number theory course, one of the most popular courses in the program. The course is one of sentimental value for me, as it is where I was first exposed to number theory, over two years ago. The course gave me an opportunity I would not have had at my home institution and I hope that by finishing the course notes, I will make the course more accessible to American undergraduates so that they can gain access to the same Hungarian treatment of the subject that I found so enthralling.

2. Development of Analytic Number Theory

Before discussing the specifics of my project and research, I would like to give the reader some background by explaining some of the fundamental aspects of analytic number theory. Some topics included are the development of sieve theory, the Green-Tao theorem on primes in arithmetic progressions, and the results of Goldston-Pintz-Yıldırım on small gaps between primes. Some terms from higher mathematics are used without explanation if understanding the specific concept is not necessary.

2.1 Origins of the Discipline

Mathematical analysis provided a foundation for many disciplines that emerged in the last two centuries. First developed as a method to rigorously define calculus, elementary proofs in analysis have a distinctive flavor and language, concerned mainly with limits of functions and sequences. However, the techniques of mathematical analysis spread into other disciplines quickly. Its power would become apparent even when applied to one of the most ancient disciplines, number theory.

Number theory is, in general, the study of whole integers and their properties. It is considered one of the oldest and most beautiful disciplines in mathematics, with seminal results which date back millennia still being studied. The proof of the infinitude of primes is considered one of the earliest results of the discipline and some of the concepts introduced by the ancient sieve of Eratosthenes would later be revitalized to provide the basis for some of the most advanced modern concepts in the field.

The disciplines provide a natural balance for each other, though the reason is hard to discern at first. Number theory deals with the discrete. Counting primes and analyzing their properties seems somehow incompatible with the continuous foundation of mathematical analysis. However, the natural regularity that the primes offer serves as a perfect avenue by which to apply the principles of analysis. While a function which counts primes under a given magnitude may be an incontinuous step function, when the average is taken, it follows a 'somewhat'[1] continuous pattern. Since the prime counting function allowed such regularity, Dirichlet capitalized on the rich theory mathematical analysis had to offer in order to prove what would serve as the foundation for analytic number theory. Dirichlet's theorem [1], states that for any x and y which share no common factors, there are infinitely many primes congruent to x modulo y.

Dirichlet's proof not only states the infinitude of such primes, but also that the sum of the reciprocals of such primes tends to infinity in a similar fashion to a well-known proof of the existence of infinitely many primes. The proof uses the concepts of the Riemann zeta function and related L-functions heavily, tools which are still at the core of the discipline. Such functions are somewhat unique in all of mathematics. While they are easy to define, their properties are still mysterious, after over a century of study. Their structure and some very basic properties are completely unknown and they represent one of the most mystifying classes of functions in mathematics.

The next major achievement in the discipline was the proof of the celebrated *prime number theorem*. Proved by Hadamard and de la Vallée Poussin in 1896 [1], it gives an asymptotic for the number of primes under a given magnitude. The proof uses methods pioneered by Euler and Dirichlet and relies on showing a lack of zeros of the Riemann zeta function on the extremal line of the critical strip. While the specific technical details of the proof are not important, one should note that the methods used in the proof are highly analytical.

2.2 Developments in the 20th Century

Even with these landmark achievements, some of the greatest problems in number theory remained unanswered at the dawn of the new century. Fermat's Last Theorem taunted mathematicians around the world and the twin prime conjecture (the existence of infinitely many pairs of primes with a difference of two) still loomed menacingly, as did the seemingly immovable Goldbach conjecture (that every even integer greater than two is the sum of two primes). The final two problems would provide a basis for some

1 The notion of 'somewhat' is characterized in analytic number theory as an error term, and is a rigorously developed concept.

of the most exciting developments in analytic number theory, and while they are still without proof, related statements have given great insight into the structure of prime numbers. In particular, the most succesful attacks, mounted through sieve methods, provided a springboard for breakthroughs in many related problems.

One of the most important techniques developed in analytic number theory in the last half-century is sieve theory. The basic idea behind it is simple and elegant. The theory owes its foundation to the ancient Greek mathematician Eratosthenes. One defines a simple set, usually all numbers under a certain magnitude. One then attempts to sift out of this set, all numbers other than the ones under analysis, usually primes of a certain form. As an example, consider the aforementioned sieve of Eratosthenes. The goal of the sieve is to find the number of primes under a certain magnitude. In order to find this, one considers the set of all numbers up to the designated threshhold and calculates how many of them are divisible by two. Next, one subtracts this number from the size of the total set, in effect, sifting out the even numbers and leaving the size of the set of odd numbers under the designated magnitude. The process is continued for each number below the square root of the desired level. The sieve does not quite work, however, in getting a good estimate for the number of primes under a given magnitude, due to technical limitations and can only sieve out numbers with small prime factors.[2]

One of the greatest results of the discipline came from the work of the Hungarian mathematician, Alfréd Rényi concerning the large sieve. The large sieve was first developed by the Russian mathematician Yuri Linnik. Alfréd Rényi further developed the concept of the large sieve in order to prove one of the most celebrated results in number theory, that there exists a constant *K* such that every even prime number can be written as the sum of at most *K* primes [1]. Rényi's result was the first that proved a weak version of the Goldbach conjecture and is one of the first steps towards a proof of one of greatest unsolved problems in mathematics.

The Chinese mathematician Chen Jungrun also proved many problems related to the Goldbach and twin prime conjectures with the same tools. Using sieve methods, he proved that there are infinitely many primes p, such that $p+2$ is either a prime or a semi-prime (a product of two primes), a statement closely related to the twin prime conjecture. Similarly, he showed that every sufficiently large even number is either the sum of two primes or a prime and a semi-prime, closely related to the Goldbach conjecture [2, 3].

2.3 Recent Discoveries

The last fifteen years have proven to be some of the most productive in the history of number theory. Andrew Wiles' proof, using concepts from elliptic curve theory, shook the world. In addition, many works further contributed greatly to our understanding of the structure of prime numbers. Some of the most notable results in analytic number theory include the Green-Tao theorem on arithmetic progressions and the Goldston-Pintz-Yıldırım results on small gaps between prime numbers.

The question of primes in arithmetic progressions has been a constant source of inspiration for new and exciting problems. Arithmetic progressions alone provide a stunning number of questions, many related to Szemerédi's proof [4] of the Erdős–Turán conjecture [5] and Erdős' conjecture on arithmetic progressions. Each of these three Hungarian mathematicians tried to discern necessary conditions to place on a set of integers to guarantee that the set contains arbitrarily long arithmetic progressions. Szemerédi's theorem gives necessary results when dealing with the density of a set. Erdős' conjecture takes the statement a step further, asking whether the divergence of the sum of the reciprocals of a set of integers implies that the set contains arithmetic progressions of arbitrary length.

It is a well known result that the sum of the reciprocals of primes diverges. The Green-Tao theorem [6] demonstrates Erdős' conjecture in the case where the set is taken to be the primes. It is a great theorem with very deep consequences. Considering that the largest known arithmetic progression of primes to this day is only of about twenty terms, the statement about the existence of arbitrarily large progressions is quite striking. Another nice result dealing with arithmetic progressions of primes comes from Antal Balog, a researcher at the Rényi and co-adviser to my project. His result (praised as 'beautiful' in the paper proving the Green-Tao theorem) shows that there are arbitrarily large sets of primes such that the average of any two primes in the set is prime. The most significant result to my project, however, is the result of Dan Goldston, János Pintz and Cem Yıldırım concerning small gaps between primes [7].

2.4 Small Gaps Between Primes

The recent results concerning small gaps between primes are not only exciting findings, but they followed quite a turbulent path to discovery. With the original findings published by Dan Goldston and Cem Yıldırım which showed that one can find arbitrarily big pairs of successive primes whose difference is arbitrarily small when compared to their logarithm, the mathematical community was very excited. Not only had they improved the best known ratio of the difference to the logarithm, but they showed that it could be taken as small as one desires, the best possible result.

But there was a major setback. After the proof had been released and was in the process of being scrutinized by the global mathematics community, a mistake was discovered. However, through the expertise of Professor Pintz of the Rényi Institute, a way to correct the mistake was

2 Eratosthenes' sieve fails quickly when trying to sieve out all such primes as one can only find exact counts for how many numbers are divisible by a number *d* when the number *d* is fixed, otherwise, an estimate must be used. The error term in this estimate encapsulates the main term as we continue sifting, rendering the method only useful in developing asymptotic estimates for numbers with only large prime factors.

found and the celebrated result stood. The improvement was such a big step forward, that many related problems' results were improved trivially by these new findings. It is on improving results related to some of these problems through these new methods that my research has focused.

3. Research, Classes and Exchange

In this section, the focus will be my project and my activities as a visiting scholar. I will talk about my time at the Rényi Institute and as a student as well as about the notes I am completing.

My stay in Hungary has allowed me to develop greatly as a mathematician with attentive advisers and a wonderful work environment. I would be hard pressed to exaggerate, or even do justice to the hospitality the Rényi Institute provided me and the patience my advisers have demonstrated as I learn this new branch of mathematics. While most of my personal research is not complete, I would like to take this opportunity to give some background on my host institution and the Budapest Semesters in Mathematics program, my second affiliation.

3.1 The Alfréd Rényi Institute

Founded by the Hungarian Academy of Sciences in 1949, the Alfréd Rényi Institute of Mathematics has housed one of the most prominent research staffs in mathematics since its inception. Temporary and permanent members have included an extremely high portion of prominent Hungarian mathematicians and it provides a home base for an extraordinary collection of contemporary researchers.

The Institute, housed in the heart of the city, has provided an excellent center from which to conduct my project. My personal lodgings have been turbulent and there were a few occasions during which if I did not have access to the technical offerings of the Institute, my work would have suffered a major setback. They provided excellent physical accommodations and access to an expansive library, where I could be confident that any source material I needed would be readily available. I recommend the Rényi Institute as a host for future Fulbright Grantees without reservations and hope that the great interchange between the Commission and the Institute is maintained.

3.2 Learning a New Field

While number theory has always been a great passion, it had not been a primary focus of mine until this year. My personal research had been mostly restricted to applications in cryptography. While I am not sure if analytic number theory will be my permanent focus, I decided to spend a year conducting research in number theory because it is such an exciting field. Referred to as the *Queen of Mathematics*, number theory used to be a subject that seemed so far removed from applicability that this 'uselessness' was a testament its beauty. Why else would generation after generation of mathematician hurl themselves at it with such vigor if no practical application to their work could ever be found? However, with the advent of electronic communications and the need for security, the Queen found another avenue through which she would compete for attention.

In modern times, number theory has great practicality. It serves as a basis for innumerable cryptographic algorithms and one can be certain that practically any virtual transaction one makes owes a great debt to the works of Euler, Fermat and Gauss. The notion of quadratic residuosity has served as one of the first utensils for the theory of zero-knowledge protocols and discrete logarithms proved to be the backbone of the first non-classified public key encryption algorithm, both central concepts in modern cryptography. With my interest in cryptography, a year to study number theory in Hungary was a perfect opportunity. I would be allowed to study alongside some of the most prominent modern mathematicians in a style which I might never have been exposed to in the United States in a very relevant discipline.

3.3 Independent Study and Classes

Serving as my advisers, Professors Antal Balog and András Biró have done more to introduce me to higher mathematics than anyone in my academic career. Since the beginning of the academic year, I have had weekly meetings with Professor Biró where we have discussed recent advancements in number theory, ranging from Kannan Soundararajan's recent work in moments of the Riemann zeta function to topics relevant to my research such as Helmut Maier's famous matrix method (which was used in a follow-up to Goldston-Pintz-Yıldırım's first paper to improve it by a linear factor).

In addition to these weekly meetings, in the first semester, I took two classes, one offered by the Rényi Institute's Gergely Harcos on *Elementary Methods in Prime Number Theory* and a second, offered by the Budapest Semesters in Mathematics program, taught by Professor Balog, *Topics in Number Theory*. Both classes served as a good way to acquaint myself with these new topics. Professor Harcos' presentation of Postnikov and Romanov's elementary proof of the prime number theorem was a highlight. I am including a similar proof in the notes for Professor Szabó's class.

In the second semester, I shifted my focus from classes and study to research. I took a second class offered by Professor Harcos entitled *Modular Forms and L-Functions*, which, while a little more removed from the focus of my study, proved to be a very rewarding class that gave insight into some of the biggest recent discoveries in mathematic. It was during this time that I also decided to take a part in the Budapest Semester in Mathematics program to encourage their goal of educational exchange.

3.4 Mathematics and Educational Exchange

I participated in the Budapest Semesters in Mathematics program, first, two years ago, as a student. It was my study abroad experience as an undergraduate and one of the primary motivators in my decision to return to Budapest. It delivered a semester heavily focused on advanced mathematics, in a beautiful new culture, with instruction by some of the most devoted teachers I had encountered. It was as a student that I met Professor Dezső Miklós, who served as my first contact when deciding to apply for a Fulbright grant for study in Hungary. He was excited to hear I was interested in returning and put me in touch with both Professors Balog and Biró. I owe a great debt to his generosity in helping me arrange the specifics of my project as well as being a very reliable source of advice.

Budapest Semesters in Mathematics owes its creation to the mathematical legends, László Lovász, László Babai, Vera Sós, and Pál Erdős. In operation for more than twenty years, it provides an opportunity for English speaking students to take courses offered primarily by professors from Eötvös Loránd University and the Rényi Institute. It boasts chances for the students to learn, among other topics, number theory and combinatorics in Hungary, one of the countries most dedicated to their study. These classes frequently feature the highest enrollment and both offer multiple sections. As I looked for an opportunity to contribute to the educational exchange mission of the Fulbright Commission in a mathematical setting, Professor Balog suggested I speak to Professor Csaba Szabó, one of the introductory number theory instructors, about helping him finish compiling the course notes for his class.

3.5 Number Theory, Hungarian Style

The above title is the subtitle for the course notes for Professor Szabó's course.

I was enrolled in Professor Szabó's class as an undergraduate and was so excited by his teaching style that I also enrolled in his *Galois Theory* course, a topic far removed from what I saw as my academic focus. His teaching style is, as an understatement, unique. There are not many professors of mathematics who introduce the topic of quadratic residues with shouts and yells, who think throwing chalk at the board is an acceptable substitute for pointing and who knowingly put erroneous information on the board to keep students attentive. A few years ago, a student had begun compiling notes for the course, but as she was taking her first course in number theory at the time and was a full time student, the notes are incomplete and in need of technical polish. Professor Szabó and I have begun working together to finish them. I attend weekly lectures and every week, deliver him my update of the typed notes along with my hand written notes. We try to capture a conversational tone with occasional anecdotes from the class thrown in for variety.

The way the material is delivered is very different from most traditional undergraduate courses in number theory. The course begins with distinguishing the concepts of irreducibility and primality, which is often neglected in such courses. This introduction is considerably more algebraic than most, as these concepts, while interchangeable when discussing prime numbers over the integers, are two very important and separate concepts in algebra. In addition, the course focuses heavily on problem solving, after all, half of the course hours are dedicated problem sessions.

Problem sessions are a concept regrettably absent in most American institutions. In his class, every Thursday, Professor Szabó will distribute a long sheet of problems, walk to the podium and exclaim several numbers. These will be the first problems that the class attempts as Professor Szabó makes his round, observing the students' attempts at solutions and giving them advice. This gives students a chance for personal interaction with the teacher as well as an opportunity for the professor to get a better understanding of the class' grasp of the course material. The absence of these sessions from American classrooms is a shame. Perhaps the reason for the absence is the pressure such scrutiny places on students; having a professor peering over one's shoulder every week is certainly not the most relaxing environment in which to learn. However, I found the experience very compatible with my own learning style and think it would be a good way to supplement courses for willing students.

4.1 Results and Conclusions

This section is dedicated to the current status of the project and the direction it will be continued in during the end of the grant period and the upcoming summer.

My research has been productive, however, as our results are not finalized, I have not included them in this report. They are related to small gaps between groups of primes and use a modification of the Goldston-Pintz-Yıldırım sieve in a related setting. Both advisers have expressed optimism about results and as soon as the project is finished, I will make the work available. There are very subtle points in modifying the argument, which may take a substantial amount of time to complete.

The course notes are still in their infancy, as I began the project in the second semester of my grant period. The first three chapters are virtually complete with about half left, corresponding to the current state of the lectures. I have inserted several appendixes to the notes, one on number theory in public key cryptography, corresponding to my interest in the subject, as well as one on an elementary proof of the prime number theorem. The proof of the prime number theorem that is being used is due to Hedi Daboussi and is important in that it is the earliest that does not use Selberg's Lemma. It is a very nice proof that was given to me by Professor Balog, which I have been translating from French, as I have not found a readily available English translation. I think that using this proof is a good choice because it is unique in relation to Selberg's Lemma and is one that may not be available otherwise. I have also included heuristics for the first elementary proof of the prime number theorem.

I hope to continue my academic relationship with both my advisers and Professor Szabó long after I leave Budapest. If our ideas bear fruit, it is likely that my advisers and I will continue to apply them to similar problems. I see my experience as a Fulbright Scholar as having affected my educational experience in a very substantial way and do not know of a way to demonstrate my gratitude. I hope to continue interacting with the Commission in the future and hope Hungary remains a popular host country for students of mathematics.

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Egy amerikai az Amerikai Kuckóban

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There are four American Corners in Hungary. My placement is at the American Corner in Pécs where I take on numerous roles. This paper describes my roles and responsibilities at the American Corner in Pécs. I pay particular attention to advising-related activities and the neccessitiy for me to seek opportunities to advise students at the Amereican Corner. I also decribe the academic and cultural programs that I helped the American Corner in Pécs present during the 2007-2008 academic year. Finally, I enlisted the help of my American Corner and Fulbright colleagues from the other American Corner cities to give their input and feedback on what an American grantee can do for an American Corner. The objective of this paper is to share my experiences so that future Fulbright grantees to Hungary can know how they can better interact with American Corners in Hungary.