Appendix A

List of American Autobiographical Literature

From The Best American Essays of the Century. Ed. Joyce Carol Oates and Robert Atwan. New York: Houghton Mifflin Company, 2000.

Angelou, Maya. "I Know Why the Caged Bird Sings." 342-357.

Bourne, Randolph. "The Handicapped." 57-70.

Ehrlich, Gretel. "Solace of Open Spaces." 467-76.

Herr, Michael. "Illumination Rounds." 327-341.

Kingston, Maxine Hong. "No Name Woman." 383-94.

Momaday, N. Scott. "The Way to Rainy Mountain." 313-18.

Oates, Joyce Carol. "They All Just Went Away." 553-63.

Rodriguez, Richard. "Aria: A Memoir of a Bilingual Childhood." 447-466.

Twain, Mark. "Corn-pone Opinions." 1-5.

Wright, Richard. "The Ethics of Living Jim Crow: An Autobiographical Sketch." 159 -70.

From Literature and Society: An Introduction to Fiction, Poetry, Drama, Nonfiction. Ed. Pamela Annas and Robert Rosen. New Jersey: Prentice-Hall, 1994.

Kovic, Ron. From "Born on the Fourth of July." 1059-69.

Whitecloud, Thomas. "Blue Winds Dancing." 1336-41

Wright, Richard. "The Man Who Went to Chicago." 858-82.

Kingsolver, Barbara. High Tide in Tucson: Essays from Now or Never. USA: Harper Perennial, 1996. 1-16.

Michie, Gregory. Holler If You Hear Me. New York: Teachers College Press, 1999. 1- 12.

Sandburg, Carl. "Chicago." http://www.carl-sandburg.com/chicago.htm. 7 April 2006.

Sedaris, David. Me Talk Pretty One Day. USA: Little Brown, 2000. 153-65.

Selzer, Richard. From "Confessions of a Knife" Modern American Memoirs. Ed. Annie Dillard and Cort Conley. USA: Harper Perennial, 1996. 100-08.

Appendix B

List of Suggested Writing Prompts

How does where you're from or your family's heritage influence who you are? Reflect on your geographic, ethnic, cultural or religious background.

What are some unsaid rules in your life (as a woman or man, as a student or teacher, as a young person, as a Hungarian, as a member of whichever group with which you identify)?

What do you want outsiders to know about your culture? Write about your culture.

Write about a time you had to stand up for a belief when most people didn't agree with you.

Whose voices are part of multicultural Hungary? Who is a Hungarian?

Bibliography

Hideg, Éva and Erzsébet Nováky. "The Future Orientation of Hungarian Youth in the Years of the Transformation."World Futures Studies Federation. 1997. **http://www.wfsf.org/pub/publications/Brisbane_97/**HIDEGNOV.pdf

Raphael, Taffy, Laura Pardo and Kathy Highfield. Book Club: A Literature-Based Curriculum. Massachusetts: Small Planet Communications, 1997.

---- "Book Club Workshop: Learning About Language and Literacy through Culture." Ciera. April 1999. **http://www.ciera.org/library/archive/1999-04/abs-online-99-04.html** In J. Many (Ed.), Instructional practices for literacy teacher-educators. Mahwah, NJ: Erlbaum. 39-49.

Modular Symbol Algorithms, Computational Number Theory, and the Millennium Problems

Stephanie Jakus

The University of Michigan, Ann Arbor
2074 East Hall, 530 Church Street,
Ann Arbor, MI 48109-1043
http://www.math.lsa.umich.edu/
Stephanie.Jakus@gmail.com

Eötvös Loránd University
1117 Budapest, Pázmány Péter sétány 1/C
www.elte.hu
Adviser: Árpád Tóth

As one of the Millennium Problems, the solutions to which carry a prize of one million dollars a piece, the Birch Swinnerton-Dyer conjecture, since its introduction in the early 1960's, has remained both a fundamental unsolved problem in algebraic number theory and one of the most challenging problems of the twenty first century. My work on the Fulbright fellowship in computing cohomology groups using the modular symbol method is not in the direction of proving the Millennium Problem, but in implementing the tools of computational number theory that have developed in the past twenty years of progress on the conjecture, and in the direction of expanding these tools for new uses. The mathematics behind the algorithms in this project lies in the intersection of algebraic number theory, homology and cohomology theory, complex analysis, and algebraic geometry, and is well explained in the order of its development using the history of the Birch Swinnerton-Dyer conjecture.

One of the seven Millennium Problems of the Clay Mathematics Institute in Cambridge, Massachusetts is the Birch Swinnerton-Dyer conjecture, named after two British mathematicians, Bryan Birch and Peter Swinnerton-Dyer, who first formulated the conjecture. The conjecture relates the number of infinite order basis elements of the group of rational points on an elliptic curve to what is called the L-function of the curve. As one of the Millennium Problems, the solutions to which carry a prize of one million dollars a piece, the Birch Swinnerton-Dyer conjecture, since its introduction in the early 1960's, has remained both a fundamental unsolved problem in algebraic number theory and one of the most challenging problems of the twenty first century. The history of the problem is noteworthy, as the predecessor to the Birch Swinnerton-Dyer conjecture, the Taniyama Shimura theorem led to one of the most well-publicized results in number theory, the solution of Fermat's Last Theorem, while the development of the mathematics instigated by work on the conjecture has led to new error-correcting codes from the algebraic geometry of elliptic curves and an improved method for factoring integers based on elliptic curves over finite fields.

My work on the Fulbright fellowship in computing cohomology groups using the modular symbol method is not in the direction of proving the Millennium Problem, but in implementing the tools of computational number theory that have developed in the past twenty years of progress on the conjecture, and in the direction of expanding these tools for new uses. The mathematics behind the algorithms in this project lies in the intersection of algebraic number theory, homology and cohomology theory, complex analysis, and algebraic geometry, and is well explained in the order of its development using the history of the Birch Swinnerton-Dyer conjecture. This essay, in the words of Benoit Mandelbrot is, "all preface from beginning to end," and is aimed at a general audience without omitting technical terminology, but also without developing it to any extent. The reader is referred to [3],[8], or [12] for details about the mathematics.

The history of the relationship between modular forms and elliptic curves that lead to modular symbol algorithms and to the partial solution of the Birch Swinnerton-Dyer conjecture, begins with such mathematical luminaries as Srinivasa Ramanujan, Felix Klein, and Henri Poincare. Srinivasa Aiyangar Ramanujan (1887-1920), the self-taught Indian number theorist who is credited with over three thousand theorems and had incredible insight into the relevance of modular forms in number theory, studied a particular modular form, the cusp form during the beginning of the twentieth century [10]. At the same time Felix Klein (1849-1925) and Henri Poincare (1854-1912) studied automorphic functions and Klein summarized his work on automorphic and elliptic modular functions in a four volume treatise [6],[7].

A modular function is a meromorphic function on the complex upper-half plane that is invariant under the action of certain groups of matrices on the upper-half plane, while a modular form is holomorphic on the upper-half plane union a point at infinity with the same invariance. The terms holomorphic and meromorphic from complex analysis refer to the existence of a power series representation for the function at all points on which the function is defined or on all but a discrete set of points respectively.

In the early part of the nineteenth century, Louis Joel Mordell (1888-1972), an American born mathematician who worked for most of his life in England, made contributions to the theory of modular forms by using what is now know as Hecke operators to prove one of Ramanujan's conjectures [9]. This was followed by the introduction of the L-function by Erich Hecke (1887-1947), and his research on the properties of the algebra of Hecke operators, two crucial steps in the direction of the Birch Swinnerton-Dyer conjecture [5]. Hecke operators play a role as averaging operators on the space of modular forms, and L-functions are functions with two inputs, an elliptic curve and a number from the complex plane. An elliptic curve is a polynomial of the form $y^2=f(x)$, where f(x) is a polynomial with degree three, or the highest power of x being x^3. It wasn't until the end of the 1950's, however, that the extent of the relationship between modular forms and the theory of elliptic curves was hypothesized.

The connection between modular forms and elliptic curves appeared in 1955 as Yutaka Taniyama's conjecture that all elliptic curves over the rationals are modular. By modular elliptic curve, Taniyama meant that the elliptic curve could be associated with a modular form in the following manner: being holomorphic, every modular form has a Fourier transform (a special kind of power series representation in terms of sine and cosine) and Taniyama conjectured that the sequence of coefficients in the Fourier transform corresponds to a sequence formed by counting the number of solutions to the equation of an elliptic curve modulus a prime for each prime number. Taniyama worked with Goro Shimura through 1957 on refining the conjecture, but in 1958, for reasons that were allusive even to Taniyama himself, he committed suicide. Shortly after, his fiancee also committed suicide because she had vowed to Taniyama never to part from him [14]. For a long time the conjecture was not even recognized as being correct, let alone of significance.

It was not until 1980 that the German mathematician Gerhard Frey suggested that the Taniyama Shimura conjecture implied Fermat's Last Theorem. Fermat's Last Theorem is the statement that it is impossible to find integer solutions to an equation of the form of the Pythagorean theorem, but with exponents, or degree, greater than two. In the seventeenth century, Pierre de Fermat had scribbled a note in the margin of a math book he was studying, saying that he had discovered a marvelous proof of this theorem, but the margin did not contain enough room for the proof. Many speculate that he could not have found such a proof, and, in any case, he never wrote it down. The conjecture remained unproven for three hundred fifty seven years. In the early nineteen nineties, Ken Ribet of the University of California, Berkeley proved that the Taniyama Shimura conjecture would indeed prove Fermat's Last Theorem, and in 1999 Andrew Wiles of Princeton University proved the Taniyama Shimura conjecture [13].

In 1993, Wiles gave a lecture on his

progress on the Taniyama Shimura conjecture to the Issac Newton Institute at Cambridge University, and, in 1995, Wiles submitted a manuscript of what he thought was a proof of the conjecture to Inventiones Mathematicae, one of Springer-Verlag's journals. A team of six was organized by Barry Mazur of Harvard University, editor of Inventiones Mathematicae, to review Wiles' proof and included Ken Ribet, Nick Katz of Princeton University, and Richard Taylor of Harvard University. The proof of the theorem depended on something called an Euler System, but the review team found an error with the use of the Euler System, and so Wiles was left to correct his proof. He invited Richard Taylor, who had formerly been his Ph.D. student, to return to Princeton where they worked together, and Taylor eventually found a way around using the Euler System, thus completing the proof. The final version of Wiles' proof was one hundred eight pages and a complementary paper published by Taylor on the techniques used to finish the proof added another nineteen pages [13].

Wiles's proof of the Taniyama Shimura conjecture was a major breakthrough for the Birch Swinnerton-Dyer conjecture because in 1990 and 1991 Victor Kolyvagin of The Graduate Center at CUNY and Karl Rubin of Stanford University had made significant advances on proving the Birch Swinnerton-Dyer conjecture, but these advances applied only to those rational elliptic curves that were modular. The proof of the Taniyama Shimura established that all rational elliptic curves are modular, thus advancing the scope of the earlier proofs [11].

Since Wiles' proof numerous books have been written on the subject of Fermat's last theorem, the most popular of which is Simon Singh's book, "Fermat's Enigma: The Epic Quest to Solve the World's Greatest Mathematical Problem." While Fermat's Last Theorem may not be the world's greatest mathematical problem, efforts to prove the theorem have certainly led to the development of algebraic number theory. Much of the theoretical mathematics that was developed to prove Fermat's Last Theorem is contained in such books as Yves Hellegouarch's, "Invitation to the Mathematics of Fermat-Wiles," and perhaps the definitive text on the subject, though requiring a lot of mathematical background, is Gary Cornell, Joseph Silverman, and Glenn Stevens,' "Modular Forms and Fermat's Last Theorem." [12]

In addition to the theory behind Fermat's Last Theorem, the field of computational algebraic number theory developed to test conjectures related to the theorem. One of the first algorithms written in computational algebraic number theory was that of Bryan Birch and Peter Swinnerton-Dyer when they tested the Birch Swinnerton-Dyer conjecture on the large EDSAC computer at Cambridge in the early 1960's. Since then, a number of mathematicians have written programs to compute invariants of elliptic curves. One of the most well known references for data on elliptic curves is the 1975 Antwerp Tables, formally known as "Modular Functions of One-Variable.

IV.," published by Springer-Verlag, which contains the proceedings of the International Summer School on, "Modular functions of one variable and arithmetical applications." The Antwerp Tables published Birch and Swinnerton-Dyer's results as well as the those of other mathematicians who had been computing invariants of elliptic curves such as A. O. L. Atkin [12]. The Antwerp Tables were the first site for publishing and disseminating the previously uncollected results of these mathematicians. In the 1990's Henri Cohen of the Université Bordeaux I, also the inventor of the Pari computer algebra system designed for fast computations in number theory, with Nils-Peter Skoruppa of the Universität Siegen and Don Bernard Zagier, Skoruppa's doctoral advisor, made extensive tables of elliptic curves but never published them in a book [12].

In 1992, and again in 1997, J.E. Cremona of the University of Nottingham published a book, "Algorithms for Modular Elliptic Curves," with the Cambridge University Press. The second chapter of the book, "Modular Symbol Algorithms," was the basis for my Fulbright project. Cambridge University Press decided not to republish Cremona's book after the 1997 edition, and so Cremona put the book on the web, and it became a well known tool and reference for algebraic number theorists wishing to write algorithms to compute the modular symbols of elliptic curves [3]. The second chapter of his book, "Modular Symbol Algorithms," gives a detailed outline of the steps needed to write an algorithm to compute what is called a modular symbol.

Just as modular forms can be defined on the upper-half plane, so can they be defined on Riemann surfaces that arise from the upper-half plane by considering a subsection, a so called fundamental domain, that is formed by taking equivalence classes of points in the half plane under a the action of a matrix group on the plane where two points in the plane are equivalent if there is a group element that takes one to the other. Modular forms on these hyperbolic surfaces are elements of the first cohomology group of the surface, a topological invariant of the surface. The i-dimensional cohomology groups and their dual homology groups measure the same property of a space, which is intuitively the number of i-dimensional rooms in a space. The equivalent to modular forms in cohomology, in homology are called modular symbols, and these play an important role in computing elliptic curves over the rationals.

In the chapter, "Modular Symbol Algorithms," Cremona focuses on computing modular symbols of the hyperbolic surface that arises from considering the upper half plane under the action of gamma zero, one of the modular groups. Though he wrote code in the programming language Algol68 to implement his algorithms, in his book he gives an overview of the mathematics behind computing modular symbols, and does not give an explicit algorithm for computing them [13]. While programming in standard computer languages is perhaps optimal in terms of efficiency, it is also time consuming to write code and find algorithms to compute standard mathematical operations.

In the years since Cremona first wrote his book, a wealth of so called computer algebra systems, software programs with many built in mathematical operations and settings, were developed to aid mathematical research. Two of the most famous of these computer algebra systems, Mathematica and Maple, are used and sold widely in the United States. The newer competitor, MuPad, developed by the MuPad Research Group at the University of Paderborn in Germany using the earlier commercial software as prototypes, is virtually unheard of in the United States, is very similar to the well known systems, a bit easier to use, and available to students and researchers in it's most user friendly form for shareware prices or with a more primitive interface for free download off of the internet. While other computer algebra systems such as Henri Cohen's PARI, mentioned above, are also available as freeware, and much faster for computations in number theory, computing and writing algorithms in the general computer algebra systems is easier and more versatile.

Therefore, I implemented the mathematics in Cremona's, "Modular Symbol Algorithms," in MuPad by writing a loop that computes modular symbols for gamma zero over a prime modulus for a given set of Hecke operators. The annotated loop is as follows:

```
/* N IS THE LEVEL FOR GAMMA ZERO */
N:=11:
/* CONSTRUCTOR IS THE FUNCTION THAT WILL GENERATE THE ARRAY OF COEFFICIENTS OF
COSET REPRESENTATIVES*/
CONSTRUCTOR:=DOM::MATRIX(DOM::RATIONAL);
/* HERE WE WRITE THE MATRICIES THAT CORRESPOND TO THE COEFFICIENTS IN THE ABOVE
ARRAY */
/* WE WON'T USE THEM UNTIL THE HECKE PROCEDURE */
FOR K FROM 1 TO N DO
    M(K):=MATRIX([[1,0],[K-1,1]]);
END_FOR:
M(N+1):=MATRIX([[0,-1],[1,0]]):
/* HERE WE BEGIN THE PROCEDURE COEFFICIENT, THE INPUT OF WHICH ARE THE ENTRIES OF
A MATRIX */
/* AND THE OUTPUT OF WHICH IS THE COEFFICIENT ARRAY */
COEFFICIENT:=PROC(A,B,C,D)
BEGIN
A:=CONSTRUCTOR(1,N+1);
/* HERE WE COMPUTE THE PARTIAL FRACTION CONVERGENTS FROM THE FIRST MODULAR
SYMBOL */
/* THE TECHNIQUE IS OUTLINED IN CREMONA CHAPTER 2 */
IF C=0 THEN A[1]:=A[1]+1; ELSE X:=OP(CONTFRAC(A/C),1);
    Q:=ARRAY(-2..NOPS(X)-1);
    Q[-2]:=1; Q[-1]:=0; Q[0]:=1;
    FOR I FROM 1 TO NOPS(X)-1 DO Q[I]:= X[I+1]*Q[I-1]+Q[I-2];END_FOR;
    FOR I FROM -1 TO NOPS(X)-1 DO
        IF Q[I-1] MOD N=0
          THEN K:=N:
          ELSE K:=(-1)^(I-1)*Q[I]*Q[I-1]^(-1) MOD N;
        END_IF;
        A[K+1]:=A[K+1]+1;
    END_FOR;
END_IF;
/* WE COMPUTE THE PARTIAL FRACTION CONVERGENTS FROM THE SECOND MODULAR
SYMBOL */
IF D=0 THEN A[1]:=A[1]-1; ELSE Y:=OP(CONTFRAC(B/D),1);
    QBAR:=ARRAY(-2..NOPS(Y)-1);
    QBAR[-2]:=1; QBAR[-1]:=0; QBAR[0]:=1;
    FOR I FROM 1 TO NOPS(Y)-1 DO QBAR[I]:= Y[I+1]*QBAR[I-1]+QBAR[I-2];END_FOR;
    FOR I FROM -1 TO NOPS(Y)-1 DO
        IF QBAR[I-1] MOD N=0
          THEN K:=N:
          ELSE K:=(-1)^(I-1)*QBAR[I]*QBAR[I-1]^(-1) MOD N;
        END_IF;
        A[K+1]:=A[K+1]-1;
    END_FOR;
END_IF;
A;
END_PROC
  PROC COEFFICIENT(A, B, C, D) ... END

/* HERE WE BEGIN THE PROCEDURE HECKE, WHICH, COMPUTES THE HECKE EIGENVALUES */
HECKE:=PROC(P)
BEGIN
/* HERE WE LIST THE HECKE OPERATORS */
MATRIX:=():
FOR S FROM 0 TO P-1 DO
    T(S):=MATRIX([[1,S],[0,P]])
END_FOR:
T(P):=MATRIX([[P,0],[0,1]]):
/* HERE WE COMPUTE THE HECKE MATRIX */
FOR T FROM 1 TO N+1 DO
    ADD:=CONSTRUCTOR(1,N+1);
    FOR S FROM 0 TO P DO
        A:=(T(S)*M(T))[1,1];B:=(T(S)*M(T))[1,2];C:=(T(S)*M(T))[2,1];D:=(T(S)*M(T))[2,2];
        ADD:=ADD + COEFFICIENT(A,B,C,D);
    END_FOR;
MATRIX:=MATRIX.CONSTRUCTOR::TRANSPOSE(ADD);
END_FOR;
H:=CONSTRUCTOR::TRANSPOSE(MATRIX);
LINALG::EIGENVALUES(H);
END_PROC
   PROC HECKE(P) ... END

HECKE(2)
   {-2, 0, 3}
HECKE(3)
   {-1, 0, 1, 4}
HECKE(5)
    {0, 1, 6}
HECKE(7)
   {-2, 0, 8}
HECKE(11)
    {0, 2}
```

While elliptic curves over the rationals are well understood because of the work of Wiles and his predecessors, elliptic curves over other number fields are not nearly as well understood. As part of my Fulbright project, I wrote an algorithm that decomposes two by two matrices with Gaussian Integer entries into a product of simpler matrices using a combination of the Euclidean Algorithm and Gaussian Elimination via matrix multiplication, and from this simplified a few steps to create a continued fraction algorithm for Gaussian Integers. The first algorithm that computes modular symbols has a special step that requires MuPad's built in function that computes continued fractions with real numbers as input. Unfortunately, this continued fraction function cannot be used with input from such special number fields as the Gaussian Integers.

The annotated loop for computing continued fractions for Gaussian Integers is as follows:

```
/* HERE WE BEGIN THE PROCEDURE, ENTRIES, WHICH COMPUTES THE CONTINUED FRACTION
OF A/B */
/* NOTE THAT C AND D ARE PLACE HOLDERS IN THIS ALGORITHM, THEIR PURPOSE WILL
BECOME CLEAR */
/* IN LATER ALGORITHMS */
ENTRIES:= PROC(A,B,C,D)
BEGIN
J:=1:
A:=MATRIX([[A,B],[C,D]]):
A0:=MATRIX([[A,B],[C,D]]):
NEWLIST:=[]:
WHILE NOT (C=0) DO
   Y:=RECTFORM(A/C);X:=A-(ROUND(RE(Y))+I*ROUND(IM(Y)))*C;Z:=ROUND(RE(Y))+I*ROUND
(IM(Y)):A:=C;C:=X;I:=J;J:=I+1;
        IF TESTTYPE(I,TYPE::EVEN)
          THEN B:=A;A:=MATRIX([[1,0],[-Z,1]])*B;
          ELSE B:=A;A:=MATRIX([[1,-Z],[0,1]])*B;
        END_IF;
   LIST:=NEWLIST;NEWLIST:=LIST.[Z];
END_WHILE;
PRINT(NEWLIST)
END_PROC
  PROC ENTRIES(A, B, C, D) ... END
ENTRIES(I,12,3+7*I,24+34*I)
   [0, 7 - 3 I]
ENTRIES(1+3*I,5+4*I,3+9*I,2+6*I)
    [0, 3]
```


In the process of writing these two algorithms, I learned mathematics from basic homology and cohomology theory to the introductory presentation of algebraic number theory dealing with modular forms given in Koblitz's book, "Introduction to Elliptic Curves and Modular Forms." Based on these studies and continuing an interest in the subject, I plan to attend the MSRI (Mathematical Sciences Research Institute) Summer 2006 Graduate Workshop in Computational Number Theory, "Computing with Modular Forms." The workshop is organized by William A. Stein of the University of Washington at Seattle, who worked with many of the mathematicians at the University of California at Berkeley involved in the proof of Fermat's Last Theorem, and who maintains a very useful website titled, "The Modular Forms Database." William Stein has designed a computer algebra system called SAGE, Software for Algebra and Geometry Experimentation, which contains numerous freeware computer algebra systems used today. For group theory and combinatorics it contains GAP, for symbolic computation and calculus, Maxima, for commutative algebra, Singular, for number theory, PARI, MWRANK, NTL, for graphics, Matplotlib, for numerical linear algebra, Numeric, and its mainstream programming language is Python. Learning this new software could potentially make writing algorithms to compute data for elliptic curves over other algebraic number fields much easier because it contains many more built in functions specific to algebraic number theory.

The current era has seen a rapid expansion in the techniques of computational algebraic number theory, due, in part to a great interest in the subject generated by the fame of Andrew Wiles' proof of the Taniyama Shimura conjecture and Fermat's Last Theorem, and by the high stature of the Birch Swinnerton-Dyer conjecture as one of the seven Millennium Problems of the Clay Mathematics Institute. My contribution towards the development of these computational techniques on the Fulbright program has been to implement J. E. Cremona's suggested modular symbol algorithm for gamma zero, and to begin to extend the algorithm for use over other number fields. I hope to continue with this project and learn more computational techniques at the MSRI workshop on computational algebraic number theory this summer.

[1] "Andrew Wiles." *Wikipedia*, *The Free Encyclopedia*. 27 Apr 2006, 21:51 UTC. 29 Apr 2006, 18:08 <http://en.wikipedia.org/w/index.php?title=Andrew_Wiles&oldid=50485946>.

[2] "Birch and Swinnerton-Dyer conjecture." *Wikipedia*, *The Free Encyclopedia*. 27 Apr 2006, 13:31 UTC. 29 Apr 2006, 18:11 <http://en.wikipedia.org/w/index.php?title=Birch_and_Swinnerton-Dyer_conjecture&oldid=50419010>.

[3] Cremona, J. E., *Algorithms for Modular Elliptic Curves*, Cambridge University Press, Cambridge, 1997.

[4] "Elliptic curve." *Wikipedia*, *The Free Encyclopedia*. 10 Apr 2006, 02:17 UTC. 19 Apr 2006, 20:27 <http://en.wikipedia.org/w/index.php?title=Elliptic_curve&oldid=47738788>.

[5] “Erich Hecke.” *The MacTutor History of Mathematics Archive*. 10 Apr 2006, 02:18 UTC. 29 Apr 2006, 10:11 <http://www-history.mcs.st-andrews.ac.uk/Mathematicians/Hecke.html>.

[6] “Felix Klein.” *Wikipedia, The Free Encyclopedia*. 23 Mar 2006, 14:05 UTC. 29 Apr 2006, 18:21 <http://en.wikipedia.org/w/index.php?title=Felix_Klein&oldid=45104808>.

[7] “Henri Poincaré.” *Wikipedia, The Free Encyclopedia*. 25 Apr 2006, 15:52 UTC. 29 Apr 2006, 18:21 <http://en.wikipedia.org/w/index.php?title=Henri_incar%C3%A9&oldid=50101536>.

[8] Koblitz, Neal, *Introduction to Elliptic Curves and Modular Forms*, Graduate Texts in Math. No. 97, Springer-Verlag, New York, 1984. Second edition, 1993.

[9] “Louis Mordell.” *Wikipedia, The Free Encyclopedia*. 11 Apr 2006, 10:19 UTC. 29 Apr 2006, 18:12 <http://en.wikipedia.org/w/index.php?title=Louis_Mordell&oldid=47934311>.

[10] “Srinivasa Ramanujan.” *Wikipedia, The Free Encyclopedia*. 28 Apr 2006, 13:12 UTC. 29 Apr 2006, 18:13 <http://en.wikipedia.org/w/index.php?title=Srinivasa_Ramanujan&oldid=50577070>.

[11] “Taniyama–Shimura theorem.” *Wikipedia, The Free Encyclopedia*. 18 Apr 2006, 02:36 UTC. 29 Apr 2006, 18:11 <http://en.wikipedia.org/w/index.php?title=Taniyama%E2%80%93Shimura_theorem&oldid=48939074>.

[12] W. Stein, *The Modular Forms Database*, 2004 <http://modular.fas.harvard.edu/Tables>.

[13] “Andrew Wiles.” *Wikipedia, The Free Encyclopedia*. 3 May 2006, 19:57 UTC. 3 May 2006, 20:29 <http://en.wikipedia.org/w/index.php?title=Andrew_Wiles&oldid=51410425>.

[14] “Yutaka Taniyama.” *Wikipedia, The Free Encyclopedia*. 31 Mar 2006, 17:47 UTC. 29 Apr 2006, 18:09 <http://en.wikipedia.org/w/index.php?title=Yutaka_Taniyama&oldid=46343740>.

The Hungarian School of Pianism: The Makings of a Pianist

Sandra Doan

Texas Christian University
2800 South University Drive
Fort Worth, Texas 76129
www.tcu.edu
s.v.doan@tcu.edu

Liszt Ferenc Academy of Music
Liszt Ferenc tér 8
H-1061 Budapest
www.lfze.hu
Adviser: Prof. György Nádor

Hungary has produced the greatest number of successful concert pianists per capita in the world in the last century. Those trained inside of Hungary can be immediately identified by their distinctive approach to the piano, the sound created from the piano, and their technical prowess. Several “piano schools” exist in the world today; their individual styles are often affiliated with their national histories and the character of their different peoples. To classify the unique approach to pianism in Hungary, this paper will trace how the country’s history and connection between Western and Eastern Europe has developed its ideals of piano performance, demanding both stylistic accuracy in performance practice, musical sensibility, and technical perfection. Then, it will examine the educational process of training a young pianist and why it is so effective. The last part of this paper will briefly analyze a set of pieces written by Franz Liszt, Deux Legendes. *Liszt is credited with founding the music school in Budapest, and his ideas influence the way music is taught today. By examining these compositions, this paper will also show how his musical ideas reflect those still emphasized today in the Hungarian school of pianism.*