

# The logical-philosophical basis of logical systems

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*The most general questions are what modern logic regards the logical role of compositionality, how it works in two—component logical semantics. After showing different versions of the compositionality of natural language we analyze the possible appearances of the principle of compositionality in two—component logical semantics. Finally, some of the most fundamental notions of intensional logical semantics are given in the way of maintaining the priority of compositionality concerning sense.*

## 1. Background

It is more than 125 years that a booklet by a hardly thirty-year old logician/philosopher was published. So something began. Something, which was not really a continuation, it revealed brand new horizons for thinkers. This booklet is Gottlob Frege's *Begriffsschrift, a formula*

*language of pure thought modeled on that of arithmetic*<sup>41</sup>, which occupies a special place in philosophical (and not only in philosophical) culture. 20th (and 21st)-century logic cannot exist without Frege's life-work. In the preface of *Begriffsschrift*, at the birth of modern logic, Gottlob Frege

<sup>41</sup> See Preface and Part I translated by M. Beaney from *Begriffsschrift, eine der arithmetischen nachgebildete Formelsprachen des reinen Denkens* (L. Nebert, Halle, 1879) in Beaney 1997, pp. 47-78.

writes he disregards expressing anything that has no significance from the point of view of consequence. He calls what he finds solely important conceptual content. Summing up my intentions, I can word my most general questions as what modern logic regards the logical role of compositionality, how it works in two-component logical semantics, how it structures conceptual content.

In Frege's view, one of the most important inventions of Begriffsschrift is the replacement of the subject—predicate decomposition by the functor-argument one. He wrote the following:

The very invention of this Begriffsschrift, it seems to me, has advanced logic. ... [L]ogic hitherto has always followed ordinary language and grammar too closely. In particular, I believe that the replacement of the concept subject and predicate by argument and function will prove itself in the long run. It is easy to see how taking a content as a function of an argument gives rise to concept formation. ... The distinction between subject and predicate finds no place in my representation of a judgement.<sup>42</sup> (Frege 1997, pp. 51, 53)

Are we aware of the significance of the step? How can we survey and evaluate its consequences? Before Frege it was generally accepted that grammatical structures were also logical ones, and what is more all logical structures had to prove to be grammatical ones. Having replaced

42 I use the expression 'functor' instead of 'function' in order to differentiate an incomplete expression of a language from its semantic value.

grammatical structures by structures relying on the functor-argument decomposition from the logical point of view the main question is whether there is a theoretical limit of the functor-argument decomposition. The limit can be found in the semantic mirror of the functor-argument decomposition, in semantic compositionality.<sup>43</sup> The structures which are relevant from the logical point of view (i.e. logical structures) must have been created in a compositional way.<sup>44</sup>

## 2. Compositionality

What is compositionality, what is meant by compositionality? Each of the following four formulations appears as the principle of compositionality. Different names (used in Szabo 2000) for different versions are appropriate to show small differences between them.

- The Principle of Compositionality (in a wide sense): The meaning of a complex expression is determined by the meanings of its constituents and by its structure. (Szabo 2000, p. 475)
- The Function Principle: The meaning of a complex expression is a function of the meaning of its constituents and of its structure. (Szabo 2000, p. 484)
- The Building Principle: The meaning of a complex expression is built up

43 Mihálydeák 2003 deals with the formal syntactical and semantic properties of the most general type-theoretical language relying on functor—argument decomposition.

44 Here I am speaking about the compositionality of natural language. The modern version of compositionality appeared tacitly in Frege's *Über Sinn und Bedeutung* (Frege 1892).

from the meaning of its constituents. (Szabo 2000, p. 488)

- The Substitutivity Principle: If two expressions have the same meaning, then substitution of one for the other in a third expression does not change the meaning of the third expression. (Szabo 2000, p. 490)

First of all I have to note that the principle of compositionality (and each of its versions) is referred to as 'Frege's principle' (or 'Fregean principle'). Philosophers and logicians dealing with Frege have been discussing for many decades whether Frege accepted the principle of compositionality (or at least one of its versions), and if he did, to what extent.<sup>45</sup> I don't want to analyze the whole story of the discussion, I would like to mention only two attitudes, the first from Hans Rott, and the second from Jeffrey Pelletier:

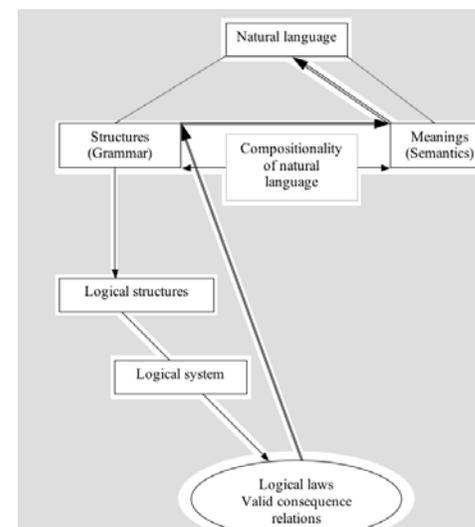
1. "The principle is certainly very Fregean in spirit ... However, as Janssen (1997, p. 421) points out, Frege does not seem to have stated compositionality as a principle in any of his writings." (Rott 2000, p. 625)
2. "Frege may have believed the principle of semantic compositionality, although there is no straightforward evidence for it and in any case it does not play any central role in any writing of his, not even in the 'argument form

45 Here I don't deal with the principle of contextuality, but as Janssen (2001, p. 115) mentions many philosophers understand the principle of compositionality as the principle of contextuality. That principle is mentioned at the beginning of Frege's *Grundlagen der Arithmetik* in the following way: "never to ask for the meaning of a word in isolation, but only in the context of a proposition" (Frege 1980, p. x).

creativity/understandability' citations." (Pelletier 2001, p. 111)

Secondly, I have to mention there is a very wide—ranging scientific discussion about the compositionality of natural language.<sup>46</sup> The place and role of the principle of compositionality in linguistics is quite controversial. It is regarded as a methodological principle, or a basic linguistic-philosophical law, or supervenience. (See, for example, Partee 1984 and Szabo 2000.)

Figure 1



In figure 1 the situation before the mentioned distinction between grammatical structure and logical structure made by Frege can be surveyed. The principle of compositionality holds and works on the level of natural language. Logical structures originated from

46 See for example the thematic issue of Journal of Logic, Language and Information (Vol. 10, 2001) on compositionality.

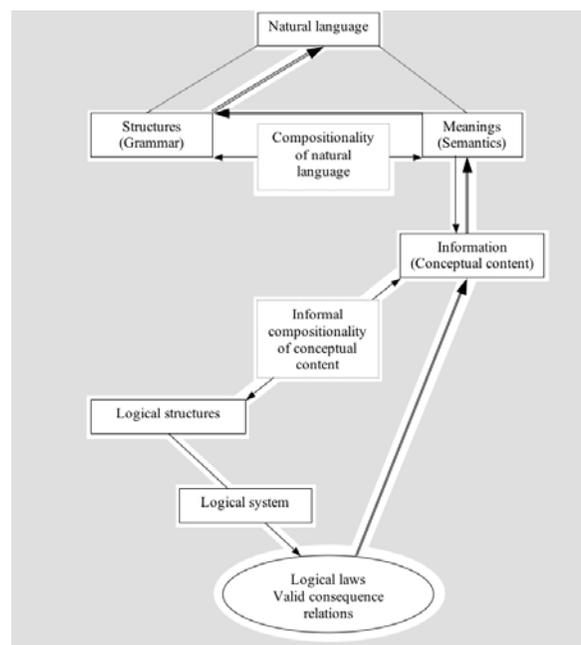
natural language directly, based only on the compositionality of natural language. Received logical laws get meaning through associated structures and compositionally joined natural language meanings. Therefore, there is no separated room for the (either informal/pretheoretic or formal) principle of compositionality concerning the logical features of natural language expressions. Natural language compositionality acts not only on the level of natural language but on the level of logical investigation as logical compositionality. It produces the main patterns of logical structures. The logical system is closed, at least in some sense, in the prison of natural language.

### 3. Compositionality and two—component semantics

Frege broke up the identity between the grammatical structure of natural language expression and its logical one. The grammatical structure may only appear as one of the possible logical structures, but an expression (or in a more sophisticated way, the expressed conceptual content) may have, and usually does have logical structures different from the grammatical structure of the expression.

In figure 2 there is a special box for informal compositionality (of information/conceptual content). What does it mean? In order to understand it we have to define the notion of semantic value, and specify what kinds of semantic values we have. Naturally, truth values

Figure 2



play a crucial role in the system of logically relevant semantic values. It can be said – regarding Begriffsschrift –, that the system of semantic values which is in the informal background is dominated by truth values in the sense that we receive the semantic values of functors from the set of truth values and the set of objects.<sup>47</sup> In his semantic writings<sup>48</sup> Frege recognized that a more flexible system of semantic values was needed to explain, for

47 As is well-known Frege considers sentences as a special type of names, and he puts the possible semantic values of sentences, i. e. truth values, into the set of possible semantic values of names, i. e. the set of objects. This unification proved problematic later, and therefore the development of logical semantics hasn't followed Frege in that sense.

48 For example: Frege-(1952a, b, c)

example, the origin of information content of identity statements. In his most famous semantic paper, *Über Sinn und Bedeutung* (Frege 1952c) he introduces an extensive version of two-component semantics, he differentiates sense and reference (or Sinn and Bedeutung).<sup>49</sup>

Taking the principle of compositionality seriously two questions may arise:

1. How should we modify the principle of compositionality (of the informal level)?
2. How does the functor—argument decomposition producing the main logical structures work with the system of semantic values of two-component semantics?

If we try to answer the first question, we will have to take into consideration that many philosophers duplicate the principle and attribute both principles (concerning the reference and the sense of compound or complex expressions) to Frege. “Crucial to Frege’s theory are a pair of principles concerning the referent and sense of complex expressions. These are the Principle of Compositionality (Interchange) of Reference and the analogous Principle of Compositionality (Interchange) of Sense. They hold that the referent or sense of a complex is a function only of the referents or senses, respectively, of the

constituent expression.” (Salmon 1994, p. 112)<sup>50</sup> Carnap was the first to attribute both versions of the principle explicitly to Frege. He wrote the following in his fundamental semantic book, *Meaning and Necessity* (Carnap 1947):

“Frege Principles of Interchangeability:

- ... First principle ... the nominatum of the whole expression is a function of the nominata of the names occurring in it.
- ... Second principle ... the sense of the whole expression is a function of the senses of the names occurring in it.” (Carnap 1947, p. 121)<sup>51</sup>

Now let us turn our attention to the second question, i.e. the behavior of the functor-argument decomposition in the case of two principles of compositionality. Anybody may think there is no problem at all, we have two different principles of compositionality, and we can use them in determining logically relevant semantic value. But which of them do we have to take into consideration, and how? In logical investigations we are interested in the truth value of a sentence, i.e. its reference. The reference can be produced by means of the first principle concerning reference, and the function occurring in it is asking for the reference of arguments. Frege recognized that in some cases the reference of the whole expression cannot be determined by means of the references of its parts. Sometimes we need to take into

49 There is no standard terminology for different semantic values. In the literature many pairs appear: sense-reference, meaning-reference, sense-meaning, sense-nominatum, sense-denotatum, meaning-denotatum, intension-extension, intension-factual value. While on the informal level I use sense and reference on the formal level I do intension and extension.

50 Quoted by Pelletier (2001, p. 88).

51 Quoted by Pelletier (2001, p. 89). The principles are similar to the function principle, which is the second version of the principle of compositionality.

consideration not only the reference of an argument but also its sense. However, at the first glance this contradicts the principle of compositionality concerning reference. How did Frege try to get rid of the problematic situation? As is well-known he differentiates between direct and indirect occurrence, ordinary (direct or customary in Frege) and indirect (oblique) reference respectively and he says if the occurrence of an argument is indirect, then its reference is its ordinary sense.

Let us pay all our attention to functors. The output of a function occurring in the principle of compositionality concerning references is the reference of the whole expression. We saw that it might depend on either the reference or the sense of its argument. Two types of functors appear:

1. If a given expression occurs directly in an expression, then the given expression can be taken as an argument and the remaining part of the whole expression, i.e. the functor concerns the ordinary reference of the given expression.
2. If a given expression occurs indirectly in an expression, then the functor (the remaining part of the whole expression) concerns the indirect reference (i.e. the ordinary sense) of the given expression.

We can say that the latter type of functors affect the (ordinary) sense of their arguments. Since we attach reference to reference, everything seems to happen according to the first principle. However, indirect reference is ordinary sense, and in order to get ordinary sense the second principle should be applied. Thus we

also have to use the second principle in determining the reference of the whole expression, we need the principle not only in the case when we are interested in the sense of the whole expression but also when we determine the ordinary reference. Frege cannot avoid the use of both principles but he didn't mention how to apply the second one, how to get the sense of the whole expression and what the connection is between the principles.

At this point we can differentiate two main types of functors and introduce the notions of Fregean intensional and Fregean extensional functor, which will prove very useful further on. Let a functor be extensional or intensional in the Fregean sense if the occurrence of its argument is direct or indirect, respectively. Of course a functor is extensional in the Fregean sense if and only if it is not intensional in the Fregean sense.<sup>52</sup> We have to note that all functors in Frege's semantic theory (called Fregean functors) affect the reference of their argument and their results are usually a given type of reference.

The subtle distinction between direct and indirect occurrence has a very problematic consequence: The reference (and therefore the sense) of an expression depends on the context in which it occurs and, of course, we have to determine not only the indirect reference of an argument, but also its indirect sense. In the Fregean

<sup>52</sup> That type of definition of the Fregean intensional and extensional functors is not usual. Generally the notion of extensional functor is defined at first, and non-extensional functors are intensional ones; the defined functors are not always Fregean.

approach reference cannot be identified with sense, thus we have to speak about the sense of an expression occurring in an indirect context and its sense can be the sense of its ordinary sense (etc.).

The next question is how the mentioned problem connected with context-dependence (or more precisely occurrence-dependence) of the type of reference can be avoided. I would like to emphasize that the problem is not the context-dependence of reference. The real problem is that in a typical fixed situation (where the ordinary reference and the ordinary sense are given) in some cases the ordinary reference is the reference and in other cases the indirect reference i.e. the ordinary sense is the reference.

Carnap recognized the problem in Frege's approach, and tried to choose another way. He characterized the differences with Frege's method as follows:

A decisive difference between our method and Frege's consists in the fact that our concept, in distinction to Frege's, are independent of the context. An expression in a well-constructed language system always has the same extension and the same intension: but [in Frege's theory] in some context it has its ordinary nominatum and its ordinary sense, in other contexts its oblique nominatum and its oblique sense. (Carnap 1947, p. 125)

The definitions of Fregean intensional and extensional functors apparently need to be modified just a little bit to get ones which are applicable to Carnap's approach. We only have to transfer the sensitivity of the type of semantic value from occurrences to functors. The result

is that those functors are extensional ones which affect the reference of their arguments in order to get the reference of the output (of the whole expression), and those functors are intensional ones which are not extensional and affect the senses of their arguments. It may seem that we are ready and the problem of context-dependence (or occurrence-dependence) is solved. We may also realize that the notion of extensional functor corresponds to the first principle of compositionality concerning reference; however, the notion of the intensional one doesn't correspond to the second principle concerning sense because an intensional functor produces not the intension but the reference of its output.

Is there any way to embed the second principle in this picture? I think we have to find the way, because the second principle is more fundamental than the first one. Sense is the most fundamental semantic value. In order that an expression may belong to a natural language, in order that it can be an expression of a given language it has to be meaningful. Accepting Kripke's approach, only proper names can be exceptions in the manner that they may be expressions of a given natural language without having sense. (Usually nobody wants to 'understand' a proper name, everybody wants to use it for referring to something.) It can be said that except for proper names there is no expression in any natural language which has reference but no sense because meaningfulness is the crucial characteristic of an expression that belongs to a given language. In two-component semantics an expression may have sense without having reference,

but we cannot understand an expression which has reference but no sense.

Relying on the two principles of compositionality we can say that the reference (nominatum) of a functor is the function which gives us the reference (nominatum) of the whole expression with the references (nominata) of different arguments, and that the sense of a functor is the function which gives us the sense of the whole expression with the senses of different arguments. We can say that every expression has sense, but what about its reference? The reference of an extensional functor is given by the first principle of compositionality directly, but we have to suppose that the reference of the argument is defined. However, for intensional functors the notion of reference cannot be defined because there is no function which gives the reference of the output from the reference of the input. What happens when an argument of an extensional functor is an intensional one? To solve this problem we have to permit semantic value gaps, which means that the sense of a functor is not a total but a partial function. Therefore, the sense of an extensional functor would be a partial function on the possible sense of its arguments that is not defined for the sense of intensional functor<sup>53</sup>.

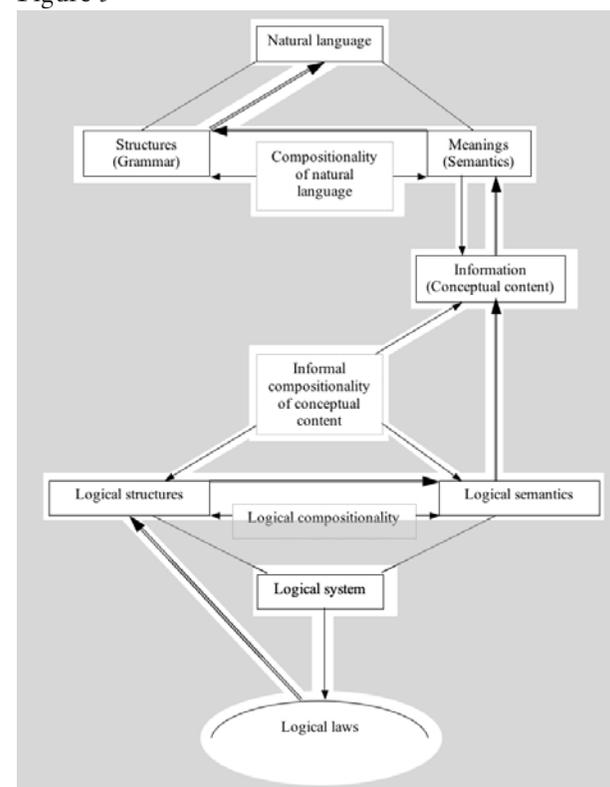
<sup>53</sup> Compare this case with the functors 'Peter believes ...' and 'Peter does not believe ...'.

#### 4. Conclusion

From the logical philosophical point of view, introducing the functor-argument decomposition and accepting its dominance enforce the informal compositionality of conceptual content which differs from the compositionality of natural language. In two-component logical semantics two different principles have to be represented. We showed that the first principle of compositionality concerning references may work in the case of extensional functors. The second one concerning senses is more general than the first. It holds for all functors. In intensional logical semantics we need both principles. It was clear that for maintaining the priority of the second principle we have to permit semantic partiality to introduce the extensional—intensional differentiation. (Figure 3)

In figure 3 a new box appears (at least in comparison with figure 2), the box of logical semantics. In order to embed the two principles of compositionality, to represent them formally, to differentiate extensionality and intensionality we have to create the whole system of logically relevant semantic values. As is well-known, possible word semantics offers a very useful possibility to treat intensionality (and extensionality) in logical semantics. Nevertheless, there are some theoretical differences in the role of the two principles of compositionality, which appear not only in the logical-philosophical background but in the formal system as well.

Figure 3



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# The Most Frequent Value Method in Groundwater Modeling

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## Abstract

*The Most Frequent Value Method (MFV) is applied to groundwater modeling as a robust and effective geostatistical method. The Most Frequent Value method is theoretically derived from the minimization of the information loss called the I-divergence. The MFV algorithm is then coupled with global optimization (Very Fast Simulated Annealing) to provide a powerful method for solving the inverse problems in groundwater modeling. The advantages and applicability of this new approach are illustrated by means of theoretical investigations and case studies. It is demonstrated that the MFV method has certain advantages over the conventional statistical methods derived from the maximum likelihood principle.*

## 1. Introduction

One of the main objectives of groundwater modeling is to determine the properly working earth models in order to adequately explain the hydrogeological observations. From the mathematical point of view, such solutions can be found

by optimization (Lee, 1999). Frequently, the inverse methods are used to determine the optimal parameter values of the groundwater models. In Earth science applications, the objective functions may have multiple hills and valleys in the